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Full Length Research Paper

Seismic performance reliability analysis for reinforced concrete buildings

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Reliability analysis assessment of seismic performance for reinforced concrete buildings was investigated in this work. This was performed through the response surface methodology in order to derive explicit expression of the failure function. Two limit states defined in terms of the total building roof displacement and the maximum inter-story drift were considered. The seismic behaviour of the building was examined by using conventional pushover analysis through finite element computations conducted by means of ZeusNL software package. Three random variables characterizing material resistance variations of concrete and reinforcement steel as well as member's sections dimensions were introduced. A complete factorial design of experiment table having three levels was used to define a finite set of data points where the failure function was evaluated, before using these results to perform identification of the building response surface model via polynomial regression. An application of this procedure was illustrated on a five story building and analysis of reliability in terms of the actual ductility coefficient was performed. Discussion was carried out about the effect on reliability resulting from the distributions of probability modelling the uncertainties affecting parameters and from using the approximate methods: Monte Carlo based sampling analysis and FORM.

Key words: Earthquake, seism, reinforced concrete buildings, pushover, FEM, reliability, FORM, Monte Carlo, response surface.

INTRODUCTION

Modern constructions must satisfy seismic design criteria which are often associated to the most adverse combination of actions with regard to building resistance under lateral loads. In order to assure safety requirements in the field of seismic design without expensive over-sizing, a major goal is to develop optimized and economic solutions (Soares et al., 2002; Quanwang, 2006; Liang et al., 2007; Koduru et al., 2007; Haukaas, 2008; Koduru and Haukaas, 2010). The pursued objective is to control risk by integrating in a rational way the effect of uncertainties which affect the applied loading, material characteristics and geometric tolerances. Reliability of structures represents a relevant tool which makes it possible to quantify the effects of these uncertainties and to calculate the probability of failure starting from the densities of probabilities associated to the random variables which are present as inputs in the problem (Hasofer and Lind, 1974; Ditlevsen and Madsen, 1996; Rackwitz and Fiessler, 1979).

This discipline not only makes it possible to calculate the probability of failure, but also to determine sensitivities associated with this probability resulting from each random variable considered separately. This gives the possibility to act in a preferential way on the most prevailing factors, when one deals for example with rehabilitation problems, in order to reach the desired reliability (Haukaas and Der Kiureghian, 2004; Haukaas and Scott, 2006).

Stochastic finite element has been introduced to account

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during analysis for the various uncertainties affecting structural model parameters. The input data for the finite element computation are dealt with as random variables to depict the uncertain variations present in the material, geometry and loading parameters. Through uncertainty propagating modeling such as Monte Carlo process, the resulting probability of response events could be computed. Finite element reliability analysis is a technique that combines stochastic finite element analysis with some performance function defining a given limit-state. The performance function depends on response quantities of the finite element analysis and is usually an implicit function of the input data. The performance function separates the data space into two regions: the safe and failure region. The probability of failure is linked to the minimum distance separating the actual design realization from the most probable failure point laying on the limit surface, also called the design point. Since the performance function is not explicitly known and Monte Carlo process is too time consuming, search of the design point is performed habitually through various approximate reliability analysis methods.

In the first order reliability method (FORM) the limit-state is approximated, at the most likely failure point in the transformed space of uncorrelated standard normal random variables, by a hyper-plane. The first coupling between FORM reliability analysis and the finite element method is found in (Der Kiureghian and Taylor, 1983). Other significant contributions have since then been presented. They include developments due to Gutierrez et al. (1994), Zhang and Der Kiureghian (1997), and Sudret and Der Kiureghian (2000).

Finite element reliability analysis using full coupling between a finite element code and reliability methods such as FORM or Monte Carlo tends however to be high computational and time consuming for practical problems. This is because, at any iteration, the limit-state function and its derivatives are to be evaluated through finite element computations.

To solve the problems associated with the finite element full coupling reliability analysis, namely simulation cost and numerical noise due to differentiation approximation, response surface based method (RSM) was introduced by (Khuri and Cornell 1996). Response surface approximations typically fit low order polynomials to a number of response simulations to approximate response. This enables us to fit the structural response such as stresses or displacements in terms of random variables for reliability analyses. The probability of failure can then be calculated inexpensively by either Monte Carlo simulation or FORM method using the fitted response surfaces.

The aim of this work is to apply the response surface based reliability analysis methodology to evaluate the seismic performance of reinforced concrete buildings. The effects of variations resulting from loading will be overlooked and the focus will be towards the effects resulting from the building structure’s geometric dimensions and from durability problems that affect the mechanical properties of reinforced concrete materials, specifically the characteristic value of the concrete resistance and the nominal yielding point of steel reinforcement. Comparison between results obtained by the Monte Carlo sampling method and the FORM method will be performed in order to see if this last is sufficient to assess seismic performance reliability of reinforced concrete buildings.

Analysis of reliability is performed in a parameterized way according to which ductility of the building structure is varied. Due to the fact that nothing makes it possible to determine with any precision the constructive provisions carried out at the time of building realization, it is very difficult to quantify the ductility in a pre-code existing building.

For a given ductility coefficient, reliability analysis is performed by taking into account the effects of three parameters assumed to be, for the sake of calculation, random variables: the sum of proportional heights of beams and columns, the concrete resistance at 28 days and the guaranteed yielding limit of steel reinforcement.

MATERIALS AND METHODS

Approximation of the performance function by a response surface model

There are several methods that can be used to approximate the performance function, also known as the limit state. One of these techniques is the response surface method (RSM). The RSM yields a simplified explicit mathematical representation of the more exact limit state and enables the evaluation of the limit state directly in terms of the random variables process inputs. In particular, it enables the drastic reduction of the computational cost by the avoidance of systematic calls for finite element computations which are typically necessary within the context of complete approaches. In full mechanical reliability coupling methods, it is necessary for the iterative evaluation of the performance function associated with any fresh set of parameters as well as the gradients by common finite differences schemes. Conversely, the RSM approximate method enables a more efficient call for the finite element code in order to gain maximal information with minimal increase in computational cost.

Moreover, the process based on RSM is more robust and needs only performing *a priori* computations by choosing pertinent trial points over the considered domain of input variables. A design of experiment table can be used and the limit state function can be derived using regression techniques to obtain an explicit analytical function which consists usually of a polynomial. The obtained approximation for the failure function by means of regression techniques is however valid only on the investigated domain of basic variables and its extrapolation outside this domain is not validated.

In the particular case of linear regression, the possibility of errors can be high. This could have significant affects on the outputs of the RSM. In order to avoid this problem, work is required with a large number of trial points and error minimization by the least squares technique is necessary. To enhance further precision of the metamodel response surface, the more accurate quadratic regression is used in this work.

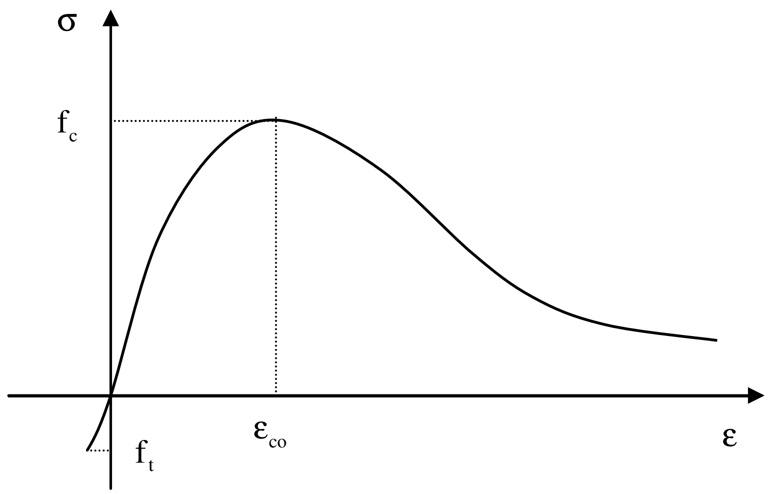

Figure 1. Uniaxial constant confinement concrete model.

Pushover simulation by means of ZeusNL software

To simulate the building response under lateral seismic loads, Zeus NonLinear (ZeusNL) software package is used, (Elnashai et al., 2008). This software provides an efficient way to run nonlinear dynamic time-history, conventional and adaptive pushover, as well as eigenvalue analysis for building structures. The modelling takes into account both geometric and material nonlinear behaviour. Concrete and steel material models are available together with a large library of elements that can be used with a wide choice of typical pre-defined steel, concrete and composite section configurations. The applied loading can include constant or variable forces, displacements and accelerations.

In the conventional pushover analysis which is used here (Hasan et al., 2002), the applied loads vary proportionally according to a predefined pattern. The post-peak response is obtained with a displacement control procedure.

Modelling static pushover under ZeusNL software requires entering material properties, section configurations, applied loadings and analysis protocol.

The concrete behaviour is assumed to be described by the nonlinear concrete model with constant (active) confinement modelling (con2) as shown in Figure 1. This enables accurate uniaxial concrete behaviour description where a constant confining pressure is assumed in order to take into account the maximum transverse pressure from confining steel. This is introduced on the model through a constant confinement factor, used to scale up the stress-strain relationship throughout the entire strain range. Improved cyclic rules were included to enable the prediction of continuing cyclic degradation of strength and stiffness, as well as better numerical stability under large displacements analysis. To use this concrete model during simulations, four parameters are required: compressive strength f_c, tensile strength f_t, crushing strain ε_0 and confinement factor. The confinement factor for confined concrete is $k = 1.2$ and for unconfined concrete $k = 1.02$.

The reinforcement steel behaviour was assumed to be that of a bilinear elastic plastic model with kinematics strain-hardening (stl1) as shown in Figure 2. This model is applied for the uniaxial modelling of mild steel. To enter this model during simulations, three parameters are required: Young's Modulus E, yield strength σ_y and strain-hardening μ.

For the regular building considered in this work, static pushover analysis was conducted under ZeusNL by taking the most adverse seismic direction and a plane gateway frame representing the building was used. Response control protocol was chosen to monitor the nonlinear analysis. This refers to the situation where the displacement of the building roof is specified by the user and is incrementally increased. The loading applied as well as the deformations of the other nodes are determined by the solution of the program.

Presentation of the case study

Pushover simulations have been conducted on a typical modern Moroccan reinforced concrete building. The selected structure has five-stories. It lays on a horizontal surface of $16 \times 18 = 288\ \mathrm{m}^2$. The inter story height is $3\ \mathrm{m}$. The building behaviour in the width direction can be represented by a five-story four-bay frame with bay length equal to $4\ \mathrm{m}$. Figures 3 and 4 depict the building elevation and plane view.

The permanent loads are $G = 5.5\ \mathrm{kN/m^2}$ and the variable loads are: $Q = 1.50\ \mathrm{kN/m^2}$. The active gravity loads are computed by taking the following combination: $W = G + 0.2\,Q$.
Design of this building has been performed using Robot office 21, where the French reinforced concrete code BAEL91, (BAEL91, 2000), was employed in conjunction with the Moroccan seismic code RPS2000, (RPS2000, 2001). The following assumptions were made: cement class: CPJ45; steel yield stress: $500\ \mathrm{MPa}$; concrete resistance: $25\ \mathrm{MPa}$; soil capacity resistance: $0.22\ \mathrm{MPa}$; structure class: II; site type: S2; seismic zone: 3; damping coefficient: 0.05; ductility coefficient: 2.

Reliability analysis

Performance functions for structural components are commonly denoted g(x), where x is the vector of basic random variables. The numerical value of the performance function distinguishes the failure state from the safe state: g > 0: safe; g = 0: limit-state; g ≤ 0 : failure.

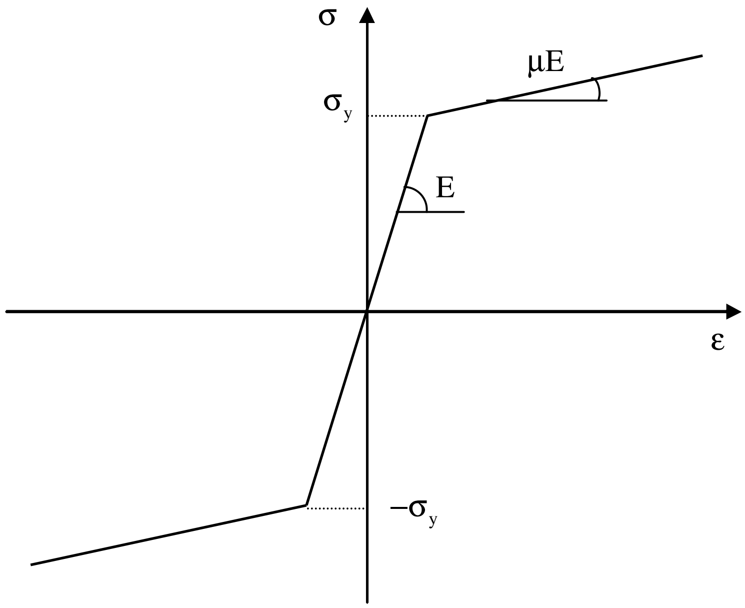

Figure 2. Uniaxial bilinear elastic-plastic law with kinematic strain-hardening modelling mild steel.

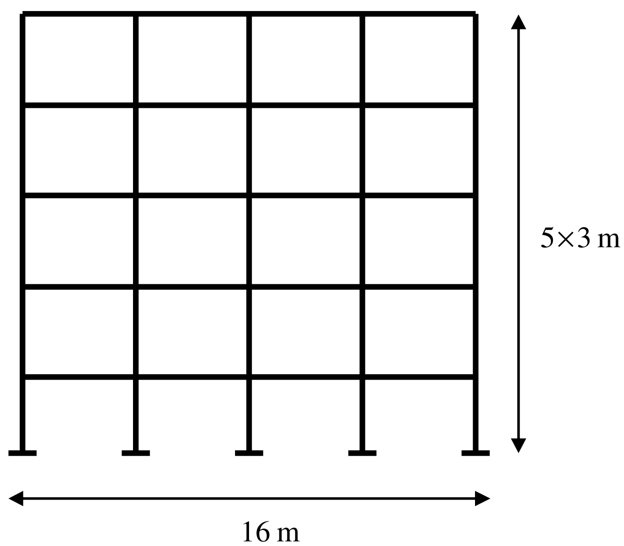

Figure 3. Vertical elevation of the five-storey reinforced concrete structure in the seismic direction.

When the uncertain response quantity exceeds the specified threshold, the performance function takes a negative value and failure is implied.

The building considered in the actual study has been designed against seismic hazard according to the Moroccan code RPS2000. The objective is to investigate seismic resistance of this building in the situation where some geometric and material properties vary from their nominal values and when all the complementary recommendations that are necessary to guarantee the chosen level of ductility were not fulfilled. This will be discussed from the point of view of reliability analysis.

The limit states chosen are those introduced by the Moroccan code RPS2000 regarding limitations of the building roof displacement and the maximum inter-story drift. The strategy followed to assess reliability of seismic performance associated to these states relies on the response surface method (RSM). A mathematical performance function is constructed in order to describe approximately the limit state of the building. This is performed by means of polynomial regressions over a set of results that are established according to a full factorial design of experiment table. Finite element static pushover computations by using ZeusNL software are performed to obtain the set of results. In our case, beam and column heights are assumed to vary proportionally and three factors have been considered:

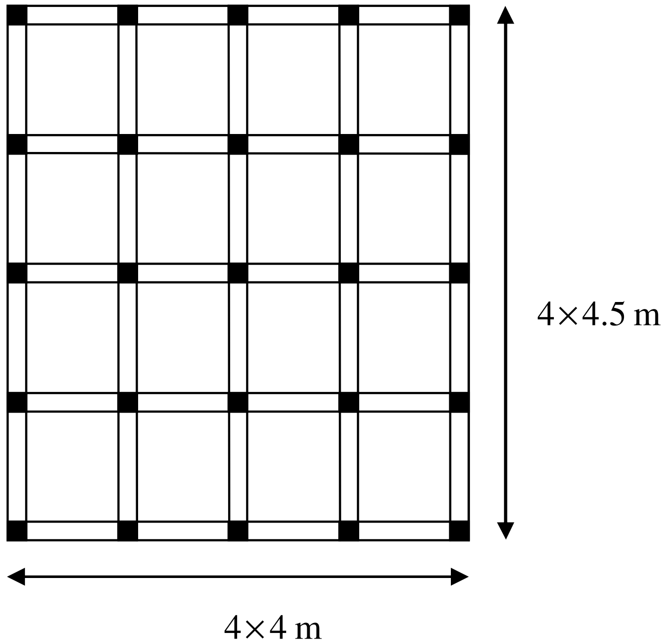

Figure 4. Plane view of the five-storey reinforced concrete structure.

Table 1. Beams and columns sections with their reinforcements.

	Section width (cm)	Section depth (cm)	Reinforcements at section bottom	Reinforcements at section top	Reinforcements at mid section
Columns	30	50	3HA14	3HA14	2HA14
Beams	25	40	6HA10	9HA10	0

i) Concrete resistance f_{c28};

ii) Steel yielding stress f_e;

iii) Sum of beam and column heights denoted by h.

When each one of the three factors f_{c28} , f_e and h is assumed to vary according to three levels, quadratic regressions can be used to interpolate the obtained roof drift and maximum inter-story drift in terms of these factors.

RESULTS AND DISCUSSION

Reliability analysis results as obtained by the response surface based methods in case of the reinforced concrete building considered in this study are compared. The limit states g_{roof} and g_{max} are assumed to suffer degradations effects which could result from durability problems affecting concrete and reinforcement's resistance or from geometric dimensions variations of columns and beams. Concrete resistance, f_{c28}, steel yielding stress, f_e, and a characteristic height of beams and columns denoted h are assumed to be random variables in this analysis. Table 1 displays dimensions of beams and columns as well as their reinforcements.

To perform nonlinear static pushover analysis, the finite element model of the building as constructed under ZeusNL software is presented in Figure 5.

Table 2 displays the obtained roof drift and maximum inter-story drift as a function of the 27 combinations of parameter values.

The identified response surfaces corresponding respectively to roof drift and to the maximum inter-story drift are readily obtained from Table 2 as:

$$\delta_{roof}(f_{c28},f_e,h) = 1.8212+9.056\times10^{-2}f_{c28}-7.306\times10^{-3}f_e-2.15h+5.2053\times10^{-4}(f_{c28})^2$$
$$+7.4714\times10^{-6}(f_e)^2+0.94111h^2-1.3102\times10^{-4}f_{c28}f_e-5.1919\times10^{-2}f_{c28}h+3.1406\times10^{-3}f_eh$$

$$\delta_{max}(f_{c28},f_e,h)=0.55981+2.5471\times10^{-2}f_{c28}-2.192\times10^{-3}f_e-0.63525h+1.5759\times10^{4}(f_{c28})^2$$
$$+2.2076\times10^{-6}(f_e)^2+0.27276h^2-3.7487\times10^{-5}f_{c28}f_e-1.4858\times10^{-2}f_{c28}h+9.3004\times10^{4}f_eh$$

The associated values of R^2 are respectively 0.987 and 0.991. This indicates that the considered quadratic.

If the roof drift is considered to be the limit state, the

Table 2. Roof drift as function of the considered case.

Combination order	f_{c28} (MPa)	f_e (MPa)	h(m)	Roof drift δ_{roof} (m)	Maximum inter- story drift δ_{max} (m)
1	28.75	537.5	1.035	0.0628	0.0170
2	28.75	537.5	0.9	0.0693	0.0186
3	28.75	537.5	0.765	0.1047	0.0282
4	28.75	500	1.035	0.0628	0.0171
5	28.75	500	0.9	0.0693	0.0186
6	28.75	500	0.765	0.1059	0.0288
7	28.75	462.5	1.035	0.0628	0.0171
8	28.75	462.5	0.9	0.0700	0.0186
9	28.75	462.5	0.765	0.3103	0.0882
10	25	537.5	1.035	0.0612	0.0166
11	25	537.5	0.9	0.0804	0.0218
12	25	537.5	0.765	0.0935	0.0251
13	25	500	1.035	0.0612	0.0166
14	25	500	0.9	0.0804	0.0218
15	25	500	0.765	0.0935	0.0250
16	25	462.5	1.035	0.0612	0.0166
17	25	462.5	0.9	0.0804	0.0220
18	25	462.5	0.765	0.0935	0.0251
19	21.25	537.5	1.035	0.0685	0.0187
20	21.25	537.5	0.9	0.0655	0.0176
21	21.25	537.5	0.765	0.0791	0.0211
22	21.25	500	1.035	0.0685	0.0187
23	21.25	500	0.9	0.0655	0.0176
24	21.25	500	0.765	0.0791	0.0211
25	21.25	462.5	1.035	0.0685	0.0188
26	21.25	462.5	0.9	0.0655	0.0176
27	21.25	462.5	0.765	0.0643	0.0178

first regressions are adequate to interpolate finite element displacement results failure function is written as

$$g_{roof}(f_{c28},f_e,h)=\delta_{c,roof} - \delta_{roof}(f_{c28},f_e,h)$$

where $\delta_{c,roof} = 0.004H$ defines the first critical collapse displacement according to RPS2000 with H the building total height ($H = 15\,m$) and $\delta_{roof}(f_{c28},f_e,h)$ the demand function as obtained here above by quadratic regression in terms of f_{c28}, f_e and h.

Considering now the maximum inter-story drift to be the limit state, the second failure function is written as

$$g_{max}(f_{c28},f_e,h)=\delta_{c,max} - \delta_{max}(f_{c28},f_e,h)$$

in which $\delta_{c,max} = 0.03\,h_s/K$ defines the second critical collapse displacement according to RPS2000, where h_s is the inter story height ($h_s = 3\,m$), K the coefficient of ductility and $\delta_{max}(f_{c28},f_e,h)$ the demand function as obtained by quadratic regression in terms of f_{c28}, f_e and h.

Two distributions of probability functions are used: Normal and Lognormal. Characteristics of these variables are given in Table 3.

The freeware reliability tools software (Rt) that was developed by (Mahsuli and Haukaas, 2010) is used here to perform reliability computations directly by using the surface response models $g_{roof} = g_{roof}(f_{c28},f_e,h)$ or $g_{max} = g_{max}(f_{c28},f_e,h)$.

For the first limit state, g_{roof}, Sampling Analysis and FORM methods are used. The obtained Hasofer-Lind reliability index β and the failure probability P_f are given in Table 4.

For the second limit state, g_{max}, associated to limitation of the maximum inter-story drift, ductility coefficient was varied in the interval $[1.1, 2]$. Both Monte Carlo based

Table 3. Characteristics of the random variables.

Variable	Mean value	Deviation ratio	Standard deviation	Probabilistic law
f_{c28} (MPa)	25	0.15	3.75	Normal, Lognormal
h (m)	0.90	0.15	0.135	Normal, Lognormal
f_e (MPa)	500	0.05	25	Normal, Lognormal

Table 4. Reliability results for the failure function g_{roof} associated to roof drift.

	Sampling analysis		FORM	
	β (Hasofer-Lind)	P_f (%)	β (Hasofer-Lind)	P_f (%)
Lognormal	-0.396188	34.5983	-0.0266167	48.9383
Normal	-0.373077	35.4545	-0.000749095	49.9701

Table 5. Importance of factors and safety coefficient as function of the ductility coefficient K, Normal distributions of probability

Ductility coefficient K	Importance of f_{c28}	Importance of f_e	Importance of h	Safety coefficient for f_{c28}	Safety coefficient for f_e	Safety coefficient for h
1.1	0.305905	0.08170	0.61240	1.06366	1.01109	1.09899
1.2	0.305185	0.08020	0.61458	1.05437	1.00938	1.08360
1.5	0.302529	0.07500	0.62241	1.02990	1.00499	1.04481
1.7	0.299572	0.06960	0.63081	1.01492	1.00240	1.02213
1.8	0.297764	0.06644	0.63580	1.00778	1.00123	1.01150
1.9	0.294553	0.06127	0.64417	0.99986	0.99998	0.99979
2.0	0.289670	0.05406	0.65627	0.99140	0.99876	0.98722

sampling analysis and FORM algorithms were used. Figure 6 gives the obtained results as function of the ductility coefficient K. Table 5 gives the importance safety coefficient for each factor.

Table 4 and Figure 6 show that results obtained by Monte Carlo based sampling analysis and FORM methods are quite different. FORM method has the tendency to overestimate the probability of failure. The difference can reach 44% in case of $K = 2$ and when the failure function is g_{max} with lognormal distributions. The difference exceeds 15% in all other cases.

It could also be seen from Table 4 and Figure 6 that when the design suffers a high probability of failure the two methods tend to be close one to each other. In this case, the sampling analysis method converges more quickly as the design point is far form the origin. But, when the probability of failure is small predictions given by these two methods are quite different. In particular, Monte Carlo based sampling analysis had not converged for $K = 2$, for which the relative difference with FORM method reaches 44% while in all other cases it does not exceed 27%. This is due to the fact that Monte Carlo based method needs a large number of iterations to converge and in the actual study this number was limited to 10000.

Table 5 shows that variability of reliability results is due essentially to concrete resistance and concrete sections, while steel resistance plays a minor role.

Table 4 and Figure 6 show that lognormal and normal distributions of probability do not give the same results even if they have the same means and standard deviations. Table 4 and Figure 6 show that the relative difference in terms of the probability of failure can reach 25% for the case $K = 2$. The difference remains between the limits 1.2 and 3.4% for the other cases. The difference is greater for the sampling analysis method as for FORM method it does not exceed 2.3%. It could be concluded that it is not sufficient to determine the statistical moments in order to perform reliability analysis and that the distributions of probability should also be properly identified. Figure 6 indicates that the probability of failure increases with decreasing level of ductility, that is, any problem regarding the required arrangements of ductility during realization of the building that prevents to have the target ductility value; here $K = 2$, would

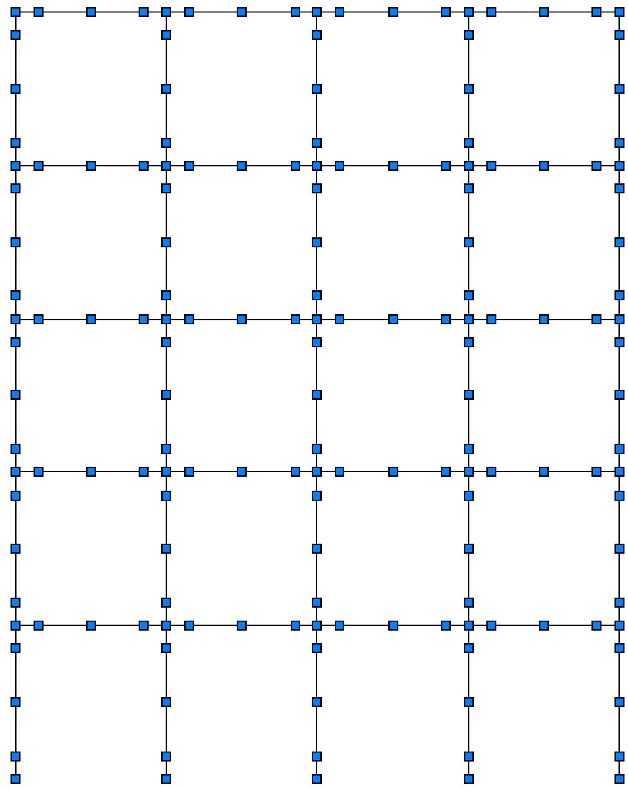

Figure 5. Finite element model of the regular building constructed under ZeusNL software package.

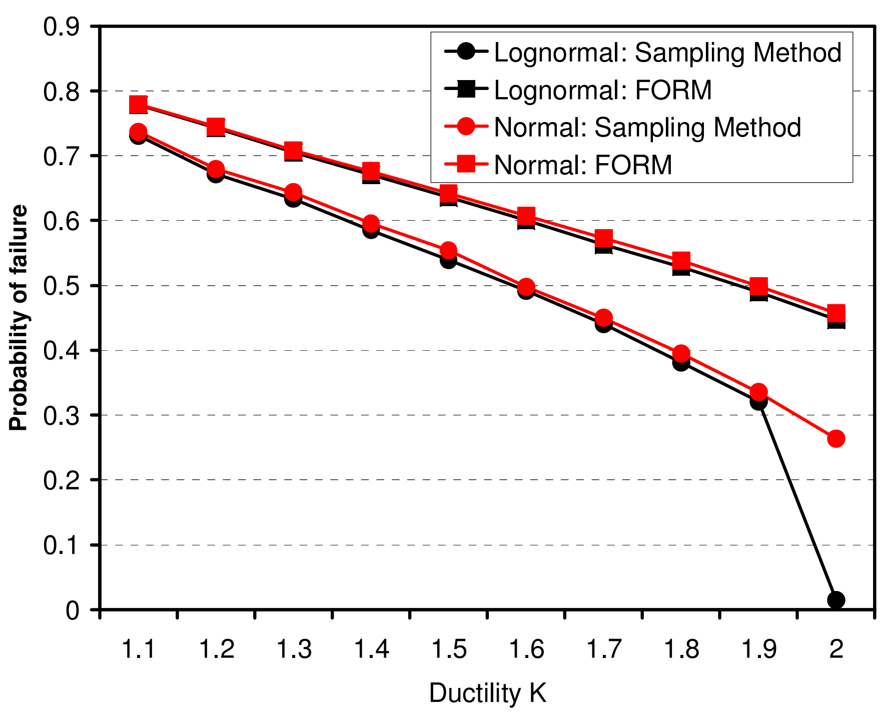

Figure 6. Probability of failure associated to the inter-story state limit as function of ductility for sampling and FORM reliability analysis methods using either Lognormal or Normal distributions of probabilities.

reduce significantly seismic performance of the building.

The uncertain variations of f_{c28}, f_e and h according to normal or lognormal distribution of probability enable simulation of seismic performance variations of the structure which is designed initially with the nominal characteristics $f_{c28} = 25\,\text{MPa}$, $f_e = 500\,\text{MPa}$ and $h = 0.90\,\text{m}$ When one integrates this variability of material performances and geometric variations, the building which was designed under the assumption of a level of ductility that is given by $K = 2$, suffers a probability of collapse. Assuming that the building is subjected to the RPS2000 seismic design load and using normal distributions of probabilities, the failure probability was found to reach a level between 26 and 46% if the design recommendations in terms of ductility $K = 2$ were fulfilled. This is quite large and indicates that reliability based engineering is of crucial importance when parameter uncertainties are expected to be high.

Further investigations are needed to get more under-standing of the complex relationships existing between the exact reliability analysis methods using full coupling and the approximate reliability analysis methods based on response surfaces. A particular problem is to study how the domain used for interpolation during derivation of response surface model can affect accuracy of results.

Conclusions

The reliability analysis concept was applied to assess, in case of reinforced concrete buildings, effect on seismic performance resulting from random uncertainties affecting basic design variables. Variations that were considered in this work included the materials and building geometry characteristics: concrete compressive strength, steel yielding resistance and member's heights. These variations could traduce in practice durability problems (corrosion, chemical attack of concrete) or material loss of structural elements.

FORM method was found to overestimate the probability of failure and hence does not enable to deal accurately with seismic reliability analysis. This was stated in the actual study through comparisons with Monte Carlo based sampling analysis method in the convergence domain of this last.

Parametric studies were performed in terms of ductility which is not really controlled in practice, particularly when the building is of pre-code type. Its influence on the probability of failure was quantified. Probability of failure was found to increase considerably as the actual ductility decreases. It was found also that the probability of failure depends on the selected distribution of probability.

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